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ABSTRACT

A common measure of performance for a binary data channel is its bit error rate. In this memorandum, the effect of a noisy reference derived from the received signal used in coherent demodulation process is evaluated. Equations are derived for data channel signal power when the data directly phase modulates the rf carrier and when the data phase modulates a subcarrier, which in turn phase modulates the rf carrier. Degradation due to the noisy reference used in reducing the received signal to baseband is shown to have the same effect regardless of whether a subcarrier is used or not.

Equations are given that provide a means of determining the bit error rate of a channel. Included in the equations for bit error rate is an expression accounting for the effects of a noisy reference.

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SUBJECT: Bit Error Rate Performance in a Coherent Binary Symmetric Channel Case 320

DATE: May 16, 1966

FROM: R. L. Selden

TM 66-2021-5

TECHNICAL MEMORANDUM

A common measure of performance for a binary data channel is its bit error rate (BER). The purpose of this memorandum is to discuss the effects on channel BER performance when the received signal is demodulated using a noisy reference signal. The telemetry data transmitted from the Apollo spacecraft (i.e. CSM and LEM) is normally transmitted in a binary symmetric channel using phase shift keyed (PSK) techniques. Some related programs such as the Apollo Lunar Surface Experiments Package (ALSEP) and Lunar Orbiter use similar transmission schemes for telemetry data.

Two modulation schemes will be considered, namely:

(1) when the PCM wave form directly modulates the rf carrier and

(2) when the PCM wave form bi-phase modulates a subcarrier which in turn modulates the rf carrier.

The discussion which follows assumes a coherent receiving system such as that used in the Deep Space Instrumentation Facility (DSIF) and the Apollo Manned Space Flight Network (MSFN). Specifically, the signal modulated onto the received rf carrier is converted to baseband by multiplying the received signal in a product detector by a reference that is derived in a narrow band carrier tracking phase locked loop. This implies that the modulation technique used must be such that a residual carrier component remains. Three techniques are generally used, these are:

(1) split-phase (Manchester) PCM directly on the rf carrier with a modulation index less than $\pm \frac{\pi}{2}$ radians,

(2) a square wave subcarrier modulated $\pm \frac{\pi}{2}$ radians by the PCM wave form modulating the rf carrier with a modulation index less than $\pm \frac{\pi}{2}$ radians and

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(3) a sine wave subcarrier modulated $\pm \frac{\pi}{2}$ radians by the PCM wave form, modulating the rf carrier with a modulation index of $\pm$ 1.84 radians or less.

Two performance parameters determine the binary channels BER performance. These are:

(1) the signal to noise ratio (SNR) in the carrier tracking phase locked loop and

(2) the received signal energy per bit.

The first parameter is a measure of the noisiness of the reference used in demodulation to baseband and, as will be shown in what follows, affects the channel BER regardless of whether the PCM signal modulates the carrier directly or modulates a subcarrier which in turn modulates the rf carrier.

It is frequently assumed that when a subcarrier is converted to baseband in a product detector, using a noisy reference, little, if any, degradation in BER appears when the subcarrier is subsequently demodulated. This is not the case, and to clarify this point we will, in the following, show that timing errors due to a noisy reference has essentially the same effect on BER regardless of whether or not a subcarrier is used.

Direct Carrier Modulation

Let the received signal be represented by:

$$s(t) = A\sin(W_c(t) \pm \theta(t)) \qquad (1)$$

where

$\theta(t)$ is a unit amplitude square wave.

The received signal is demodulated by multiplying in a product detector by:

$$r(t) = B\cos(W_{ct} - \phi(t)) \qquad (2)$$

where

$\phi(t)$ is the phase error in the loop due primarily to input noise.

The output of the detector is then:

$$d(t) = s(t)r(t) = \left[B\cos(W_c(t) - \phi(t))\right]\left[A\sin(W_c(t) \pm \theta(t))\right] \quad (3)$$

$$= \frac{AB}{2}\sin(\pm\theta(t) + \phi(t))$$

The second order terms have been ignored (assumed to be filtered).

This may be expanded as follows:

$$d(t) = \frac{AB}{2}\left[\sin(\pm\theta)\cos(\phi(t)) + \sin(\phi(t))\cos(+\theta(t)\right] \quad (4)$$

The signal power is then given by:

$$\overline{d(t)^2} = \frac{A^2B^2}{4}\sin^2(\pm\theta(t))\cos^2(\phi(t)) \quad (5)$$

Subcarrier Modulation of the Carrier

It is now desirable to determine the effect of the carrier tracking loop phase error on a subcarrier detector output when the subcarrier is detected to baseband in a product detector as was the PCM waveform above.

Let the received signal be represented by:

$$s(t) = A\sin\left[W_c(t) + M_1\sin(W_{sc}(t) \pm \theta(t))\right] \quad (6)$$

where

M_1 is the modulation index of the carrier by the subcarrier

and the reference by

$$r(t) = B \cos(W_c(t) - \phi(t))$$

Then the demodulator output is

$$d(t) = s(t)r(t) = \frac{AB}{2} \sin \left[M_1 \sin(W_{sc}(t) \pm \theta(t)) + \phi(t)\right] \quad (7)$$

This can be rewritten as

$$d(t) = \frac{AB}{2} \left[I_m\right] \exp \left[M_1 \sin(W_{sc}(t) \pm \theta(t))\right] \exp(\phi(t))$$

$$= \frac{AB}{2} \left[I_m\right] \sum_{n=-\infty}^{\infty} Jn(M_1) \exp \left[n(W_{sc}(t) \pm \theta(t))\right] \exp(\phi(t)) \quad (8)$$

Since we are only interested in the first order terms of $W_{sc}(t)$, we will evaluate at $n=\pm 1$ only

$$d(t) = \frac{AB}{2} J_{-1}(M_1) \sin \left[-W_{sc}(t) \mp \theta(t) + \phi(t)\right]$$

$$+ \frac{AB}{2} J_1(M_1) \sin \left[W_{sc}(t) \pm \theta(t) + \phi(t)\right] \quad (9)$$

Simplifying we have:

$$d(t) = AB\, J_1(M_1) \sin(W_{sc}(t) \pm \theta(t)) \cos(\phi(t)) \quad (10)$$

Now we will assume this subcarrier is demodulated in a product detector with a perfect reference. Let us represent the subcarrier demodulation reference by:

$$r(t)_{sc} = C \cos(W_{sc}(t)$$

Then the demodulator output is

$$r(t)_{sc}d(t) = ABJ_1(M_1)\sin(W_{sc}(t)\pm\theta(t))\cos(\phi(t))\ C\cos(W_{sc}(t))$$

$$= \frac{ABC}{2} J_1(M_1)\sin(\pm\theta(t))\cos\phi(t)) \qquad (11)$$

The signal component to the decision device is then:

$$\overline{d(t)}^2_{tm} = \frac{A^2B^2C^2}{4} J_1{}^2(M_1)\sin^2(\pm\theta(t))\cos^2(\phi(t)) \qquad (12)$$

Comparing equations (5) and (12) will show that the timing error caused by a noisy reference derived in the carrier tracking loop manifests itself the same way whether a subcarrier is used or not. It is well to point out that the subcarrier demodulation process is usually subject to a similar "noisy reference" phenomena, and some additional degradation could be expected if care is abandoned in the demodulator design. From this we can conclude that the timing error will degrade equally a pcm signal transmitted on the rf carrier and one that is modulated first onto a subcarrier.

Methods for predicting BER performance considering these two parameters are available in the literature [1,2,3,4]. It is of interest now to evaluate the effect of this phase error $\phi(t)$, on the channel's BER performance.

The bit error rate in a symmetric binary channel using optimum detection is well known[1] and is given by:

$$P_E = \text{erf}\left(\frac{E}{N_o}\right)^{\frac{1}{2}} \qquad (13)$$

where E/N_o = received signal energy per bit per noise in 1Hz (N_o Is a double-sided noise spectral density)

and

$$\text{erf } x = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp(-y^2/2)\, dy \qquad (14)$$

P_E can now be rewritten to include the effect of $\phi(t)$ as follows:

$$P_E = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{E/No}\cos\phi(t)}^{\infty} \exp(-y^2/2)\, dy \qquad (15)$$

This integral has been evaluated, and limited data are available on the BER performance of a binary symmetric channel as a function of the received signal energy per bit and the carrier tracking loop signal to noise ratio (when the demodulation reference is derived in the carrier tracking loop.) Viterbi has shown that the probability density function in a second order loop can be approximated, with good results by[2]:

$$p(\phi) = \frac{\exp(\alpha\cos\phi)}{2\pi I_o(\alpha)} \qquad (16)$$

where α = carrier tracking loop signal to noise ratio

I_o = Bessel function of zero order and imaginary argument

This probability density function has been expanded by Lindsey[4] to include two-way coherent communications systems. Lindsey has also published data relating BER to the carrier loop SNR and the received energy per bit [4,5]. It is not shown in reference [4] that the above probability function was used in computing the BER data presented. It can be shown, however, that when Lindsey's "two way" probability density function is used, (as in reference [4]) and the up-link carrier to noise ratio is assumed to be ∞ the phase error statistics in the ground receiver carrier tracking loop simplify to that given in equation 16 above.

If the above probability function is assumed for the phase error in the loop, the probability of error in the channel can be computed from:

$$P_E = \int_{-\pi}^{\pi} \frac{\exp \alpha\cos\phi}{2\pi I_o(\alpha)} \int_{\sqrt{E/No}\cos\phi}^{\infty} \frac{\exp\left[-y^2/2\right]}{\sqrt{2\pi}} d\phi dy \qquad (17)$$

It should be pointed out that laboratory experiments have been conducted at the Jet Propulsion Laboratory which verified the applicability of equation (17)[6]. This specific equation was evaluated by Viterbi[2] for different values of α, holding E/No constant at 4.8 (this corresponds to an ideal BER of 10^{-3} when $\alpha=\infty$). Lindsey has evaluated both the above equation and the expanded expression for performance in a two way coherent system. His data are presented in references 3 and 4.

The bit error rate data in references 3 and 4 provides not only a means to predict channel performance but also a useful tool that can be used to determine system parameters in a new design. Some of these parameters are:

(1) carrier tracking loop noise bandwidths

(2) power division of the transmitted signal between the data and the carrier

(3) feasibility of using or not using a subcarrier

(4) transmitter power required for a given bit rate (and BER) and

(5) determination of the quality of the ultimate design (i.e. how close to theoretical does the channel perform).

In summary, the following can be concluded from the above discussion of channel bit error rate performance.

(1) The BER performance of a binary symmetric channel is a function of both the received signal energy per bit $\left(\frac{E}{N_o}\right)$ and the noisiness $\phi(t)$ of the demodulating reference.

(2) The noisy reference has the same effect regardless of whether a subcarrier is used.

(3) When the demodulating reference signal is derived from a carrier tracking phase locked loop, the loop signal to noise ratio is a significant parameter in determining a channel's BER.

(4) If the signal to noise ratio in the carrier loop is low, the noisy reference determines a channel's BER.

R. L. Selden

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Attachment
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